MODELLING AND ANALYSIS OF THE MECHANISM OF HUMAN JOINT FUNCTION: EFFECTS OF ACCUMULATION OF HA MOLECULES DURING MOTION

A Synopsis
Submitted for the partial fulfillment of the requirement for the Degree of

Doctor of Philosophy
in
Mathematics

By

KAPIL SHEKHAR

Dr. Agam Prasad Tyagi Supervisor Depart. of Mathematics Faculty of Science	Prof. G. S. Tyagi Dean Faculty of Science	Prof. Ravindra Kumar Head Depart. of Mathematics Faculty of Science

DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
DAYALBAGH EDUCATIONAL INSTITUTE
(DEEMED UNIVERSITY)
DAYALBAGH, AGRA-282005, INDIA

MODELLING AND ANALYSIS OF THE MECHANISM OF HUMAN JOINT FUNCTION: EFFECTS OF ACCUMULATION OF HA MOLECULES DURING MOTION

INTRODUCTION

The mechanical behavior of the human locomotion system has become an important field of Bio-Fluid research. The knowledge of the joint loads during daily activities is furthermore of great importance for the design and testing of artificial joint replacements and provides valuable information on the interaction between the artificial implant and the bone. This study is concerned with the analysis of the mechanical behavior of joint by means of a mathematical model.

Why do human joints wear out during aging ? Perhaps a more exciting question is how do joints resist wear over a lifetime? Synovial joints of the human body (e.g., knee, hip, and elbow) are complex living biological and mechanical systems consisting of articular cartilage, bone, menisci, ligaments and synovium that allow for joint articulation and movement with minimal friction and wear. These joints are less constrained by ligamentous attachments compared to fibrous or cartilaginous joints, such as knee joints.

It was Pawaskar [39] who investigated contact mechanics of the synovial joints. However, to understand the role of interstitial fluid in contact mechanics and tribology, synovial joints allow free movement of the bones and are characterized by a synovial cavity, connective tissue and the cartilage covering each of the articulating ends of the bones acting as a bearing material [54,52,53]. For example, articular cartilages in the hip joint cover the acetabular cavity and the femoral head. This articular cartilage has remarkable lubricating properties with a very low coefficient of friction in the range of 0.001 to 0.02 [23,31,40] and low wear rates [30,40]. Though, it normally survives the life time of a person, sometimes it suffers degradation and wears due to many factors including

age, improper use like extreme sports, injury, trauma, wear and tear, and congenital diseases [55,57].

BEARINGS AND SLIDE WAYS

A wide range of conditions are encountered in rolling and sliding contacts in textile machinery. In particular, materials and lubricants have to be selected for bearings and slide ways operating over a wide speed range.

Fluid film lubrication normally represents the ideal operating conditions for bearings and slide ways. The opposing solids are completely separated by a film of lubricant whose thickness is many times greater than the heights of the surface irregularities on the solids. In this condition wear is prevented by a simple expedient of preventing contact between the solids.

If an intermittent lubrication supply system is used there is every possibility that fluid film lubrication will give way to a second well known lubrication regime; boundary lubrication. This type of lubrication implies that the solid surfaces are not continuously separated by a relatively thick layer of lubricant and that the frictional behavior of the bearing is determined by the surface properties of the solids and lubricant. The action is governed by chemistry rather than physics and its title is attributed to Hardy [18]. It frequently occurs when the continuous lubrication system described earlier is replaced by occasional lubricant injection from hand pumps or oil cans.

In boundary lubrication, the thickness of the fluid film is between 10^{-7} – 10^{-6} cm. McCutchen [25] concluded that the boundary lubrication was very important in the highly loaded articular surface while fluid film lubrication was important in soft tissue lubrication. It arises whenever the film thickness falls to the same order of magnitude as the height of the surface roughness and the size of surface- active molecules of the lubricant which may be bonded to the

solids. Additives are frequently applied to lubricants when boundary lubrication conditions are encountered. Boundary lubrication can also occur in very low- speed or highly loaded plain bearings under steady operating conditions or when insufficient lubricant is supplied to the bearing. The wear rate under boundary lubrication conditions might be reduced to a satisfactory level, but it would be considerably greater than the overall wear rate encountered in fluid-film bearings.

A considerable amount of lubrication research work is now focused upon the region of operation between the fully developed fluid-film and boundary conditions. It has been found that many of the operating conditions thought to be governed by the effects of boundary lubrication are in fact an extension of hydrodynamic action known as 'elastohydrodynamic lubrication'. If high pressures are generated within the lubricant the viscosity may rise considerably.

Elastohydrodynamic lubrication governs the behavior of highly loaded contacts of low geometrical conformity like gears and rolling-contact bearings. The elastohydrodynamic action is often capable of extending the fluid-film characteristics of a contact when the designer is unable to impose the high geometrical conformity normally associated with physical wedge bearings. In elastohydrodynamic lubrication, the thickness of the fluid film is between 10^{-5} – 10^{-4} cm. The history of elastohydrodynamic lubrication goes back to 1886, when Reynolds [45] published his famous article where he derived the differential equation describing the pressure distribution and load carrying capacity of lubrication films for journal bearings.

In hydrodynamic lubrication, the thickness of the fluid film lies between 10^{-4} – 10^{-3} cm. This is much larger than the roughness thickness. The pressure in the fluid film is generated because of relative motion of surfaces and wedge action. If the surfaces are not moving and the pressure is generated by external pumping, we use the term 'hydrostatic lubrication'. In experimental studies

[37,50] have shown clear evidence of load enhancement and friction reduction effects due to the pressure of additives.

McCutchen [26] proposed a novel type of fluid film action termed ‘weeping lubrication’. The concept of weeping lubrication is based on the assumption that cartilage is a weeping bearing and load is carried by the hydrostatic pressure of the fluid when cartilage weeps under loading, even though the liquid content of the porous cartilage would be expelled and there would be an increase in the friction coefficient if the squeeze is maintained indefinitely. McCutchen [25] showed that while weeping lubrication was important in joints, an obvious boundary lubricant was also present. He carried out various tests using synovial fluid as a lubricant for rubber against glass and careful design of the sliding surface; he eliminated any fluid film action and showed that adsorption on the surface undergoing sliding was important.

HUMAN KNEE JOINTS

Nature has provided very complex systems and novel bearings at human joints which, operates on the combined principles of boundary layer lubrication and fluid-film lubrication. However, the contribution of each form of lubrication is not clear and the physical mechanical behavior of the synovial joint is not well understood. The point at which two or more bones are connected is called a joint.

Synovial joints are the most common and movable type of joints in the human body (or in other mammals). They (freely movable joints) make up 90% of our body. Synovial joints are necessary for the basic locomotion of the human body. Knee, wrist, finger, shoulder, elbow, ankle and hip are synovial joints. The inner lining of synovial joint is called the synovium [42]. These weight bearing joints consists of two bone ends, which are covered with layers of soft poro-elastic

material called articular cartilage. The space between these two cartilage surfaces is filled with a suspension of hyaluronic acid molecules in blood plasma called synovial fluid [9,13,14]. The whole joint system is surrounded by connective tissues and ligaments, which keeps the stability and integrity of the joint during motion i.e., the whole joint capsule consists of a tough outer layer which helps stabilize the joint and an inner membrane that produces the lubricating synovial fluid [9,21].The synovial joint have high degree of geometrical conformity between their surfaces. Geometrically, the human knee joint can be represented by cylindrical surfaces. The radii of these surfaces vary from 2 to 100 cms. The general physical configuration of a synovial joint is shown in fig. 1.1.

In a human knee joint (fig.1.2), the femur and tibia, two major bones of the leg undergo relative motion during body movements but the lateral bone, the fibula, does not take part in the articulating motion, Engine and Korde [17]. It is the lower end of the femur, consisting of medial and lateral epicondyles and the upper end of the femur, consisting of medial and lateral plateaus that are covered with the articulating cartilage. Steindler [51] suggested that because of the obliquity of the femur axis, more load is supported by the lateral condyle than medial one.

The sliding speed in the joint varies from 0 to 10 cm/sec. The load support area in a human knee joint be 4.7 sq. cm [16]. The structure of the joint is such that it can support load as high as four times the body weight by keeping the coefficient of friction as low as 10^{-2} [29].

ARTICULAR CARTILAGE

Articular cartilage, also called hyaline cartilage, is the smooth, glistening white tissue, thickness varying from 0.1 to 0.7 cm, consisting of cells embedded in non-homogeneous poro-

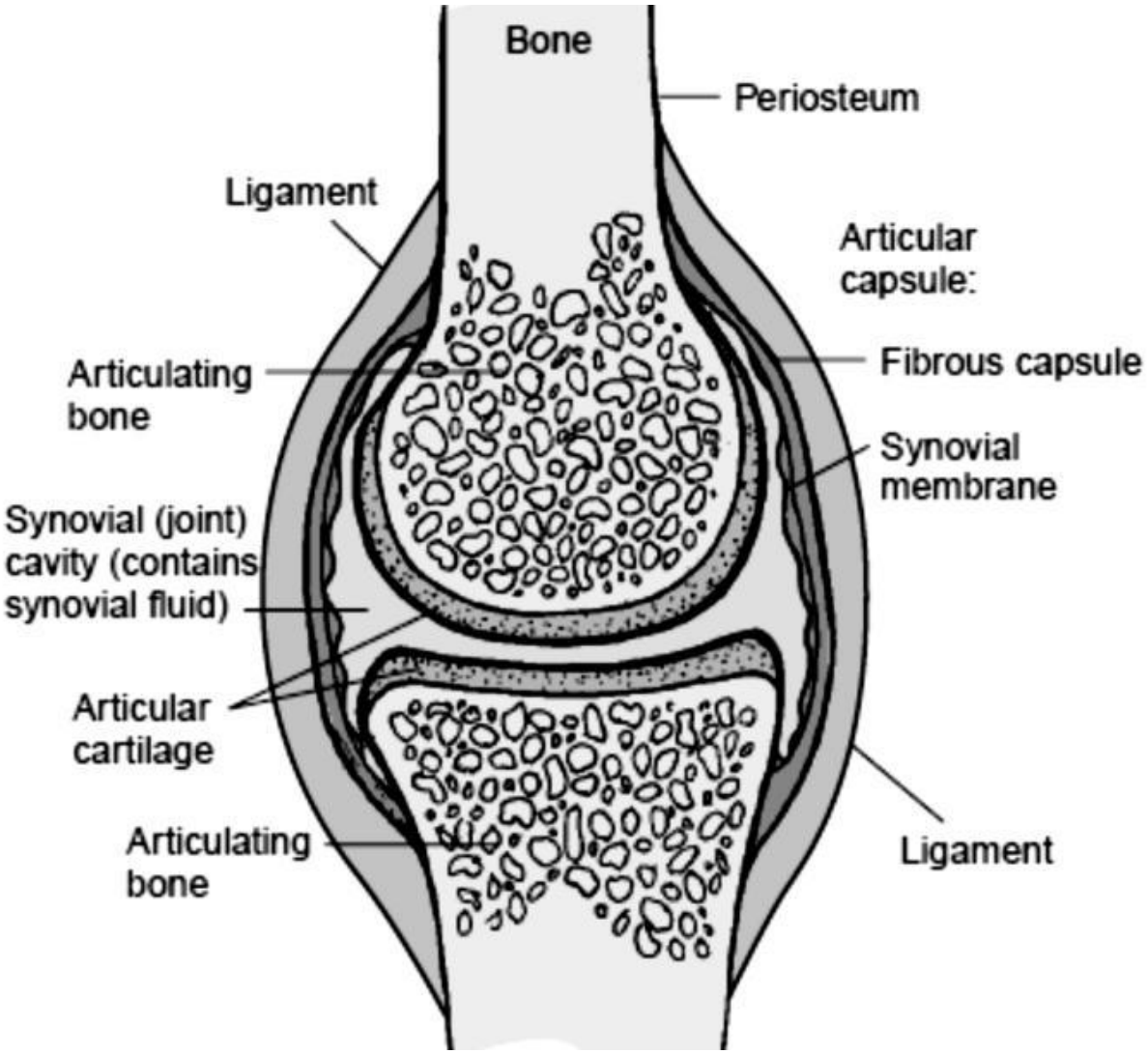

Figure 1.1: Synovial Joint

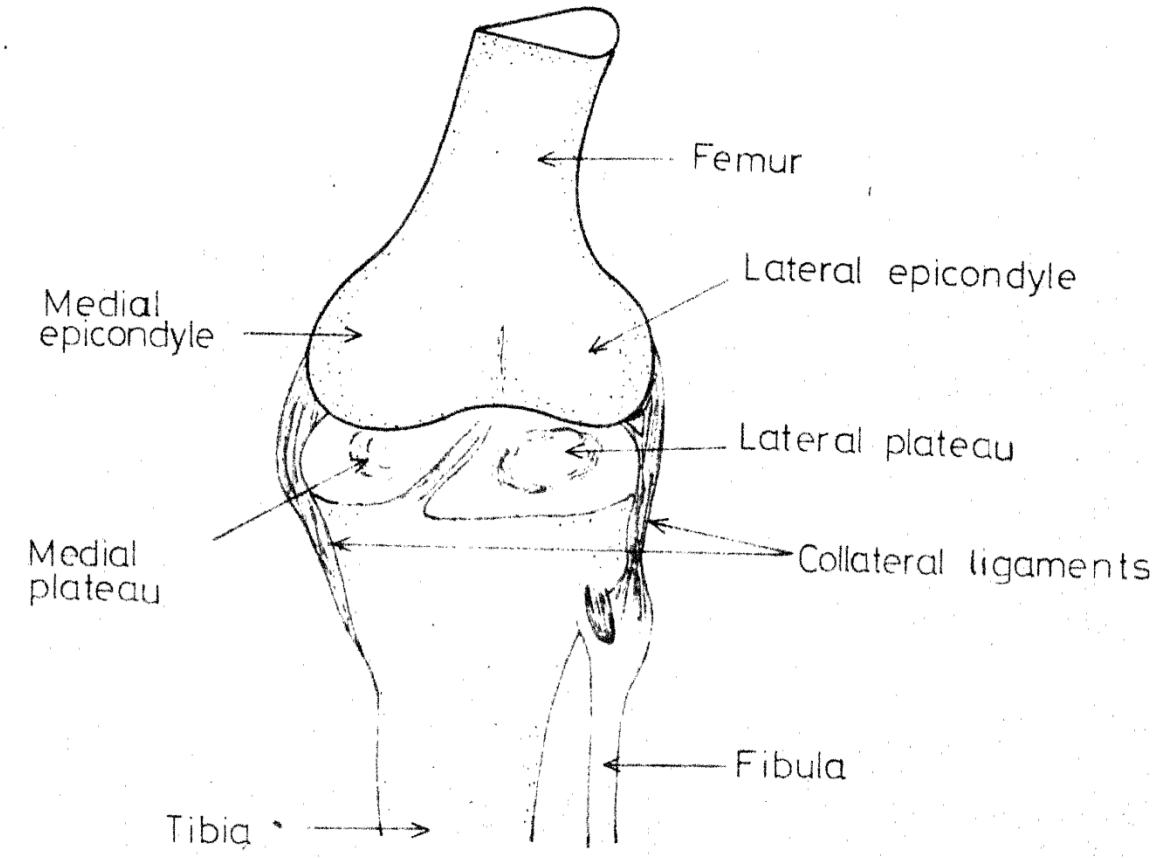

Figure 1.2: Human Knee Joint

elastic matrix of collagen fibrils saturated with aqueous fluid [1,20] and covering the surface of all the diarthrodial joints in the human body. The cartilage is permeable with pore diameter of the order 6 x 10^{-7} cm and its permeability varies with cartilage thickness. The normal coefficient of permeability near the surface varies from 3 to 9 x 10^{-13} cm^3 sec/gm and in the deep zone from 1 to 2.5 x 10^{-13} cm^3 sec/gm. It is found that the tangential coefficient of permeability is of the order 5.8 x 10^{-13} cm^3 sec/gm for a given specimen. It is noted experimentally that the tangential permeability is equal to the average of the normal permeability for given specimen.

Clarke [10] has made the scanning the electron microscope study of articular cartilage and has pointed out that it is a layered medium. The cartilage is a three layered porous medium consisting of a superficial tangential zone, a middle zone and a deep zone [16]. Articular cartilage has an incredibly low coefficient of friction which, coupled with its ability to bear very large comprehensive loads, makes it ideally suited for placement in joints, such as the knee and hip. The complex composition and architecture permits it to achieve and maintain proper biomechanical function over a majority of a human lifespan. Interstitial fluid in the articular cartilage contains water and positively charged solutes. The cartilage in the average young (25–34 years) human male and female undergoes approximately 5400 and 4700 loading cycles, respectively, during normal daily activities 13 and more than 108 loading cycles over an 80-year lifespan. Even during simple daily activities, such as walking, cartilage sustains mechanical forces several times higher than the body weight.

The cartilage is made up of extra-cellular matrix (ECM), chondrocytes and water. The interstitial water containing dissolved electrolytes is around 60 – 80% of the total wet weight [4, 23, 38]. The ECM is a reinforced structure consisting of a dense network of collagen type II fibers and

proteoglycan gel with a high degree of cross-linking. The cartilage structure is inhomogeneous, non-linear, anisotropic and exhibits time-dependent behavior [25].

The healthy cartilage supports a wide range of complex loads, as high as 7 – 9 times body weight [6] and motions at different speeds ranging from 0.06 m/s to 0.6 m/s [32] and yet is able to provide excellent lubrication mechanism over these differing and at times conflicting loading regimes. Different lubrication mechanisms such as hydrodynamic, elasto-hydrodynamic, boundary, boosted, weeping and mixed lubrication have been hypothesised to explain the remarkable tribological performance under a wide variety of operating conditions in synovial joints.

In unhealthy cartilage, these lubricating and wear properties are compromised, e.g. as in osteoarthritic cartilage. Osteoarthritis is a degenerative disease of the articular cartilage in which it gets degraded ultimately leading to a complete breakdown thus initiating bone to bone contact causing pain. The initiation of cartilage breakdown is still unknown; however, it is thought to be induced due the mechanical factors causing structural as well as biochemical changes [2,33]. There are many ways of treating Osteoarthritis and treatment generally falls into three broad categories; non-pharmacological, pharmacological and surgical treatments [8].

SYNOVIAL FLUID

Synovial fluid is a clear or yellowish dialysate of blood plasma consisting of molecules of a non-sulphated mucopolysaccharide called hyaluronic acid (H.A). In a normal joint the concentration of the hyaluronic acid is 3.5 mg/g. Long chains of hyaluronic acid give synovial fluid many beneficial rheological properties [56].The synovial fluid film forms a thin layer at the cartilage surface of the order 5×10^{-3} cm. The hyaluronic acid is a straight long chain polymer compound with molecular weight 10^6 and the molecular length of the order $5 \times 10^{-5} - 10^{-4}$ cm [16]. It may be noted that the acid molecules, normally, cannot pass through the cartilage pores

because of their large dimensions. Synovial fluid exhibits non-Newtonian behavior at low shear rates but it may behave as a Newtonian fluid at high shear rates such as in the case of jumping [20,28,35].

Its viscosity ranges from 100 P (=100gm/cm sec) at 10^{-2} sec^{-1} shear rate to 10 CP (= 0.1 gm/cm sec) at 100 sec^{-1} [13]. The hyaluronic acid with higher molecular weight $(> 17 \times 10^6)$ shows entanglement in synovial fluid, whereas lower molecular weights do not show such behaviour [13,14,17]. In solutions, the hyaluronic acid molecules believe to form a three dimensional network which occupy a solvent domain estimated at 10^3-10^4 times the volume of the polysaccharide chain itself. It is said that these networks are entangled even at very low concentrations [13,49]. The normal human synovial fluid also contains extremely low concentrations of lipoproteins and apolipoproteins, in sharp contrast to those found in plasma and a small cellular component, which is believed to be secreted by the cells lining the joint cavity, the synovial cells. Increased amounts of cholesterol and other lipids have been found in the synovial fluid of a chronic inflammatory joint disorder such as rheumatoid arthritis [21,24,41].

The viscoelastic properties that synovial fluid exhibits are particularly well suited to the joint since the fluid is highly viscous at low shear rates such as during gait, and highly elastic during high shear rates, such as in jumping or trauma. The joint's synovial fluid is at work in many complex mechanisms within the joint to maintain efficient motion. Synovial fluid is a primary component of the lubrication process. In both boundary layer and fluid film theories it is essential that synovial fluid provide necessary lubricating molecules and viscous properties, respectively. Synovial fluid and articular cartilage are closely linked in providing lubrication and changes in either will have an effect on the performance of the joint.

Healthy joints contain hyaluronic acid (HA) molecules with a molecular weight between 1 and 10 million [3,27]. In 1953 Ogston and Stanier [36] showed that HA concentration has a marked influence on the rheological properties of synovial fluid. They also postulated that it is the HA that facilitates both the static and dynamic lubrication in the joint. Through a unique experiment Ogston and Stanier [36] were also able to demonstrate that as synovial fluid is compressed it behaves in a very elastic manner up to a point at which it will no longer compress under a given load, thus preventing the two loading surfaces from coming in contact. This effect, presumably due to the presence of HA, has significant implications in the lubricating ability of synovial fluid and will be discussed in greater detail later. There are few man made materials that even approach the low coefficient of friction that has been measured between articulating cartilage. Synovial fluid further enhances the lubricating abilities of the knee joint. Synovial fluid is extremely viscous at low shear rates and extremely elastic at high shear rates. Additionally it contains lubricating molecules that contribute to even lower frictional properties. These properties allow synovial fluid to act as an ideal lubricant to prevent friction and wear of cartilage surfaces [5].

DISEASED JOINTS

In joint disease i.e. arthritis, or many forms of it, the synovium becomes inflamed and thickened, producing extra fluid containing inflammatory cells. These inflammatory cells can then attack and damage the cartilage and the bone underlying it. Although scientists researching arthritis have come a long way toward understanding how joints work, we still do not know exactly what causes arthritis. Even so, prevention, early detection and appropriate medical and surgical treatment can help most arthritis sufferers, Shekhar et. al [46].

Osteoarthritis (fig. 1.3), the most common form of arthritis, occurs when joint cartilage breaks down, exposing bone ends and allowing them to rub together. The result can be pain,

stiffness, loss of movement and sometimes swelling. Osteoarthritis is most often found in the weight-bearing joints such as the hips, knees, ankles and spine.

Rheumatoid arthritis (fig. 1.4) is the second-most-common form of arthritis, and it occurs when the immune system turns against the body. The faulty immune system causes inflammation and swelling that begins in the joint lining and spreads to the cartilage and the bone. Rheumatoid arthritis tends to affect joints symmetrically.

LUBRICATION MECHANISMS OF HUMAN JOINTS

It has been pointed out by MacConaill [22] that human joints function under hydrodynamic condition. But later several alternatives lubrication mechanisms have been laid under different moving conditions [9,22]:

a. Hydrodynamic Lubrication [22]
b. Boundary Lubrication [11]
c. Elastohydrodynamic lubrication (EHL) [21,26]
d. Weeping Lubrication [26]

In the hydrodynamic model, synovial fluid is regarded as a lubricant, which separates the articular cartilages forming the human joints. The two-cartilage surfaces form a wedge-shaped lubricant film similar to those developed in a tilting pad thrust bearing [9,11]. An alternative to this Charnley [11] proposed boundary lubrication between cartilage surfaces on the observation that hydrodynamic lubrication was not suitable under heavy load and slow motion.

In the mechanism of human joint lubrication is that the application of load caused the fluid within the porous cartilage to act as a pressurized hydrostatic bearing. The term weeping lubrication

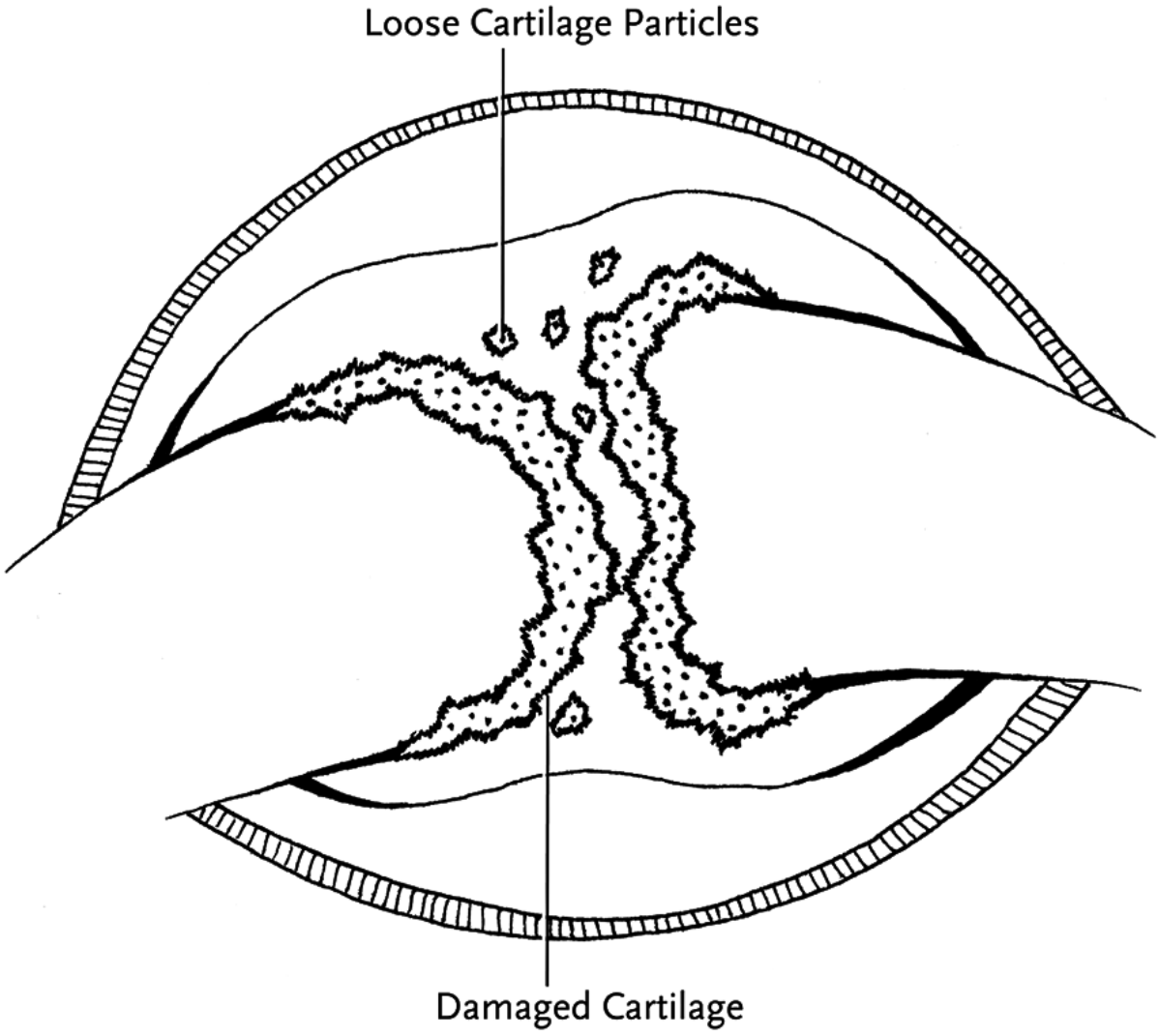

Figure 1.3: A joint with osteoarthritis

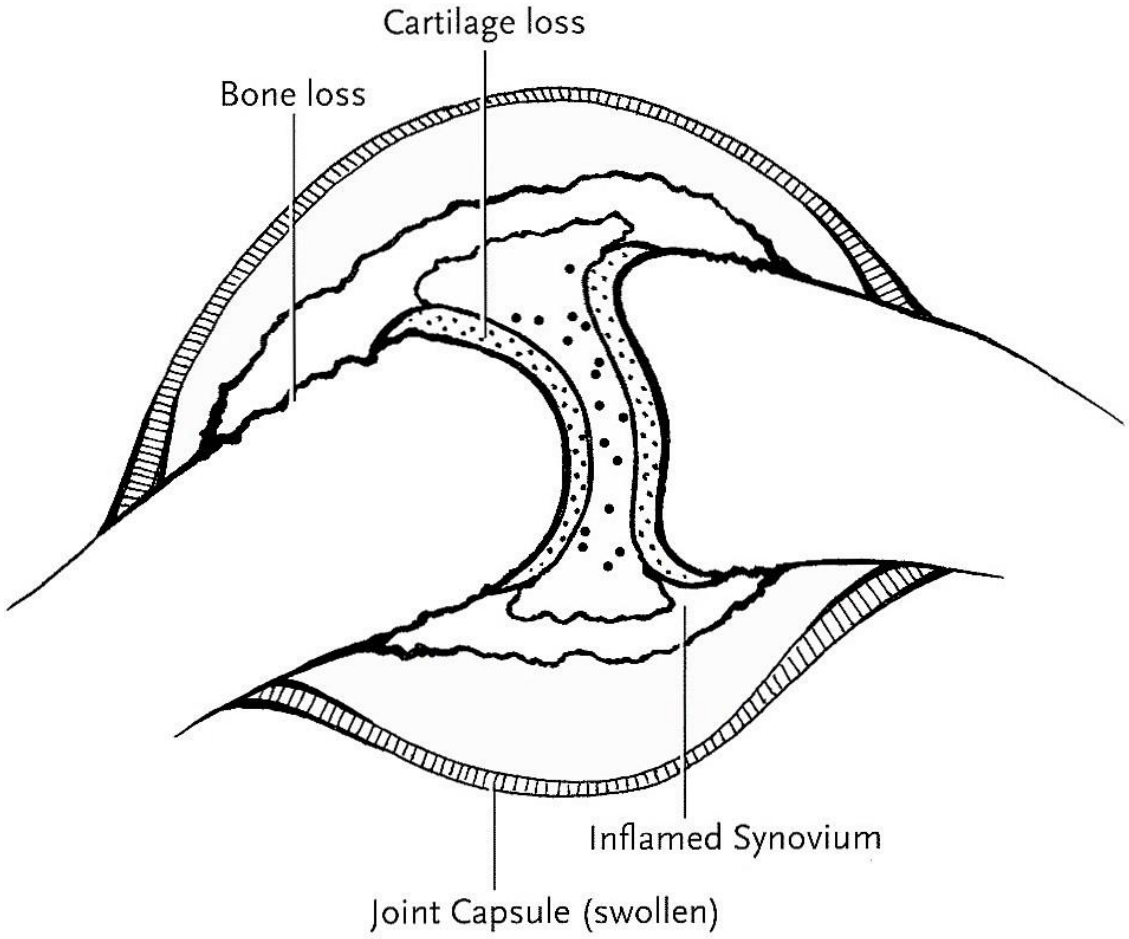

Figure 1.4: A joint affected by rheumatoid arthritis

has been introduced for this condition [24]. New support for a hydrodynamic action in joints has been provided by using the concepts of elastohydrodynamic lubrication [15]. It has also been suggested that the boundary lubrication mechanism might be influencing the surface tractions at asperity contacts on the surface of cartilage [50].

It is noted here that under highly loaded conditions during jumping, synovial fluid thickness become as small as the dimension of the hyaluronic acid molecules, then these molecules get confined and stuck between two cartilage surfaces forming a three dimensional network of porous matrix between them. During motion, the plasma part of the synovial fluid only flows through this matrix experiencing resistance to flow useful for supporting load. It is pointed out here that in the normal joint during squeezing the thickness of the cartilage matrix decreases, the aqueous fluid from the cartilage flows into the synovial fluid film almost transversely augmenting the capacity of joint to support more load. The lateral flow in the cartilage is relatively small.

PROBLEMS TO BE STUDIED

To study the characteristics of a joint depends upon many factors as mentioned below:

a. **Articular cartilage**, its porous and elastic behavior, the size, direction and the distribution of the pores in the matrix [12,48].

   Many investigators have studied the effect of porosity of bearing surfaces in the porous matrix and an appropriate Reynolds equation in the fluid film region, Chandra [9] and Dinnar [15] .As the cartilage surfaces are porous, the same concept will be used to investigate the effect of porosity in synovial joints.

   The cartilage is being considered elastic by approximating the load bearing region as parallel plates. We will study the effects of cartilage elasticity at the time of squeezing of synovial joints by considering rectangular and circular plate model, Chandra [9], Kumar et.

al. [19], Naduvinamani et. al [34], Rao et. al [44].

b. **Synovial fluid**, its viscosity, non-Newtonian viscoelastic nature, concentration of the hyaluronic acid molecules, the size of these molecules and their interaction with cartilage surface.

The concentration of the hyaluronic acid molecules (HA) depends upon whether the joint is normal or diseased. It also depends on the specific processes such as diffusion, biochemical reaction, consolidation, trapping etc. during joint movement. To determine the concentration of HA molecules in normal and disease case, we have assumed the exponential form of viscosity which depends on fluid film thickness and viscosity parameter, Chandra [9], Qvale et. al. [43], Shekhar et. al [47]. So on this basis we can accounted the HA molecules.

Shukla [49] and Chandra [9] has used power law model to envisage non- Newtonian model and also established the basic governing equation of motion for the synovial fluid in the joint cavity. We will follow the same concept to envisage non- Newtonian model.

c. **Joint motion and its Geometry**, nature and location of a joint matrix of motion, linear or oscillatory, etc.

It is noted here that the presence of HA molecules in the synovial fluid confined between two porous cartilages play very important role in the lubrication of various human joints. During motion the distribution of HA molecules in the synovial fluid changes and this causes viscosity variation of the synovial fluid in both radial and axial directions. The focus in the thesis is to study these aspects

in the hydrodynamic lubrication of human joints by considering some aspects of **Articular cartilage, Synovial fluid, Joint motion and its Geometry.**

Generally the following assumptions are made in the modeling process:

- Synovial fluid film thickness is small and is of the same order of as the mean diameter of HA molecules (network).
- During squeezing (jumping) first synovial fluid film thickness decreases followed by decrease in cartilage thickness.
- During squeezing, the tangential flow takes place only in the synovial fluid film. No tangential flow in the cartilage but only vertical flow in synovial fluid.
- HA molecules may be touched to the cartilage surface while forming three dimensional works.
- The flow in the central core of synovial fluid film follows Brinkman's model [7].

Since squeeze film mechanism prevails in human joints in case of standing position, particularly when synovial fluid thickness become as small as the dimension of the hyaluronic acid molecule, these molecules do not move, and the plasma part of the synovial fluid flows through the matrix formed by hyaluronic acid molecule, experiencing resistance and thus creating pressure gradient in the fluid, useful for supporting load. It is further noted that during squeezing process, as the film thickness of synovial fluid decreases, the concentration of hyaluronic acid molecule in the synovial fluid increases causing changes in the viscosity of the synovial fluid in both radial and axial directions. The effects of viscosity variation of synovial fluid on hydrodynamic lubrication of human joints are proposed to be studied. In the modeling process, the concept proposed by Shukla [49] is to be applied.

In the following, some of the problems mentioned below in the case of human joint lubrication are proposed to be studied by using mathematical models:

i. Squeeze film mechanism: Applicable in standing position or jumping position.
ii. Slider bearing fluid flow mechanism: Applicable in walking or running position.
iii. Radial and axial directions, longitudinal and transverse directions depending upon the geometry of the joints.
iv. Effects of the formation of two or more layers due to viscosity variation.
v. Effects of viscosity in the above cases.
vi. Effects of Non-Newtonian behavior of the synovial fluid on joint function.

It is hoped that the study presented in the thesis would be useful in curing some diseases afflicted by human joints.

REFERENCES

1. Ateshian, G. A. and Hung, C. T. (2006), "The natural synovial joint: properties of cartilage", Proceeding of the Institution of Mechanical Engineers, Journal of Engineering Tribology, Vol. 220 (8), pp. 657-670.
2. Aigner, T., and McKenna, L. (2002), "Molecular pathology and pathobiology of osteoarthritic cartilage", Cellular and Molecular Life Sciences (CMLS), Vol. 59(1), pp. 5-18.
3. Aviad, A. D. and Houpt, J. B. (1994), "The molecular weight of therapeutic hyaluronan (sodium hyaluronate): how significant is it ?", The Journal of Rheumatology, Vol. 21(2), pp. 297-301.
4. Armstrong, C. G., and Mow, V. C (1982), "Variations in the intrinsic mechanical properties of human articular cartilage with age, degeneration, and water content", The Journal of Bone and Joint Surgery-American Vol. 64(1), pp. 88-94.
5. Anadere, I., Chmiel, H., and Laschner, W. (1979), "Viscoelasticity of normal and pathological synovial fluid", Biorheology, Vol. 16(3), pp. 179-84.
6. Bergmann, G., Graichen, F., and Rohlmann, A. (1993), "Hip joint loading during walking and running, measured in two patients", Journal of Biomechanics, Vol. 26(8), pp. 969-990.
7. Brinkman, H. C. (1947), "A calculation of the viscous forces exerted by a flowing fluid on a dense swarm of particles", Applied Scientific Research, Vol. A1, pp. 27–34.
8. Carrabba, M., and Sarzi-Puttini, P. (2004), "Introduction: Osteoarthritis in the third millennium: A new era for an old disease? ", Seminars in Arthritis and Rheumatism, Vol. 34(6, Supplement 2), pp. 1-2.

9. Chandra, P. (1975), “Mathematical models for synovial joint”, PhD Thesis, Indian Institute of Technology, Kanpur.
10. Clarke, I.C. (1971), “Articular Cartilage: A review and scanning electron microscope study: 1. The interterritorial fibrillar architecture”, J. Bone Joint Surg. Br., Vol. 53 (4), pp. 732-750.
11. Charnley, J. (1959), “The lubrication of animal joints”, Proceeding of the Symposium on Biomechanics, Instn. Mech. Engrs., London, pp.12-22.
12. Deresse, A. G. and Sinha, P. (2011), “THD analysis for finite slider bearing with roughness: special reference to load generation in parallel sliders”, Acta Mech. 222, pp. 1–15.
13. Davies, D. V. (1967), “Properties of synovial fluid”, Proceedings of the Institution of Mechanical Engineers, Vol. 181(3J).
14. Davies, D. V. (1966), “Synovial fluid as a lubricant”, Federation Proceeding, Vol. 25(3), pp. 1069-1076.
15. Dinnar, U. (1976), “Two porous layers lubrication in human joints”, Annals of Biomedical Engineering, Vol.4, pp. 91-107.
16. Dowson, D. (1967), “Modes of lubrication in human joints”, Proceedings of the Institution of Mechanical Engineers, London, Vol. 181(3J), pp. 45.
17. Engine A. E., and Korde M.S., (1974), “Biomechanics of normal and abnormal joints”, Journal of Biomechanics, Vol.7, pp. 325-330.
18. Hardy, W. B. (1936), Collected works (Cambridge University Press).
19. Kumar, J. V. and Rao, R. R. (2015), “Effects of Surface Roughness in Squeeze Film Lubrication of Two Parallel Plates”, Tribology in Industry, Volume 37 (2), pp. 161-169.

20. Kobayashi, Y., Okamoto A. and Nishinari, K. (1994), “Viscoclasticity of hyaluronic acid with different molecular weights”, Biorheology, Vol. 31, pp.235-244.
21. MacConaill, M. A. (1967), “Basic anatomy of weight-bearing joints, part III”, Proceedings of the Institution of Mechanical Engineers, Vol.181 (5), pp.1-7.
22. MacConaill, M. A. (1932), “The function of intra-articular fibrocartilages, with special reference to the knee and inferior radio-ulnar joints” Journal of Anatomy, Vol. 66, pp.210-227.
23. Macirowski, T., Tepic S., and Mann, R. W. (1994), “Cartilage stresses in the human hip joint”, Journal of Biomechanical Engineering-Transactions of the Asme, Vol. 116(1), pp.10-18.
24. McCutchen, C. W. (1967), “Physiological lubrication, part III”, Proceedings of the Institution of Mechanical Engineers, Vol. 181(1), pp.1-8.
25. McCutchen, C. W. (1966), “Boundary lubrication by synovial fluid: demonstration and possible osmotic explanation”, Fed. Proc., Vol. 25, pp.1061-1068.
26. McCutchen, C. W. (1959), “Sponge-hydrostatic and weeping bearings”, Nature, London, Vol. 184, pp. 1284-1288.
27. Mensiteri, M., Ambrosio, L., Nicolas, L., Bellini, D., and O’Regan, M. (1996), “Viscoelastic Properties Modulation of a Novel Hyaluronic Acid Polymer”, Journal of Materials Science: Materials in Medicine, Vol. 7, pp. 695–698.
28. Mo, Y. and Nishinari, K. (2001), “Rheology of hyaluronan solutions under extensional flow”, Biorheology, Vol. 38, pp. 379-387.
29. Morrison, J. B. (1970), “The mechanism of the knee joint in relation to normal walking ”, J. Biomechanics, Vol. 3(1), pp. 51-61.

30. Mow, V. C., and Huiskes, R. (2005), “Basic Orthopaedic biomechanics and mechano-biology”, Lippincott Williams & Wilkins: Philadelphia, Third Edition, pp. 720.
31. Mow, V. C., and Lai, W. M., (1980), “Recent developments in synovial joint biomechanics”, SIAM Review, Vol. 22(3), pp. 275 - 317.
32. Mow, V. C. (1969), “The role of lubrication in biomechanical joints.” Journal of lubrication technology, Trans. ASME, Vol. 91(2).pp. 320-328.
33. Muehleman, C., and Arsenis, C. H. (1995), “Articular cartilage. Part II. The osteoarthritic joint”, J. Am. Podiatr. Med. Assoc., Vol. 85(5), pp. 282-286.
34. Naduvinamani , N. B. and Savitramma, G. K.(2013), “Squeeze Film Lubrication between Rough Poroelastic Rectangular Plates with Micropolar Fluid: A Special Reference to the Study of Synovial Joint Lubrication”, Hindawi Publishing Corporation, ISRN Tribology, Volume 2013, Article ID 431508, pp. 1-9.
35. Nasseri, S., Bilston, L. and Tanner, R. (2003), “Lubricated squeezing flow: A useful method for measuring the viscoelastic properties of soft tissues”, Biorheology , Vol. 40; p. 515-551.
36. Ogston, A. and Stanier, J. (1953), “The physiological function of hyaluronic acid in synovial fluid: viscous, elastic and lubricant properties”, J. Physiol., Vol.119, pp. 244.
37. Oliver, D. R. (1988), “Load enhancement effects due to polymer thickness in a short model journal bearing”, J. Non- Newtonian Fluid Mech., Vol. 30, pp. 185-196.
38. Olsen, S., and Oloyede, A. (2002), “A finite element analysis methodology for representing the articular cartilage functional structure”, Computer Methods in Biomechanics & Biomedical Engineering, Vol. 5(6): pp. 377 - 386.
39. Pawaskar, S. S. (2010),"Joint Contact Modelling of Articular Cartilage in Synovial Joints,” PhD Thesis, The University of Leeds School of Mechanical Engineering, UK.

40. Persson , B. N. J. ,(2000), “Sliding friction”, Springer-Verlag: Berlin, Second Ed., pp. 515.
41. Prete, P. E., Gurakar-Osborne, A. and Kashyap, M. L. (1995), “Synovial fluid lipids and apolipoproteins: A contemporary perspective”, Biorheology, Vol. 32(1), pp.1-16.
42. Price F.M., Levick J.R., and Mason R.M., (1996), “Changes in glycosaminoglycan concentration and synovial permeability at raised intra-articular pressure in rabbit knees”, J. Physiol., Vol. 495 (3), pp. 821-33.
43. Qvale, E. B., and Wiltshire, F. R. (1972), “The performance of hydrodynamic lubricating films with viscosity variations perpendicular to the direction of motion”, Journal of Lubrication Technology, Volume 9(1), pp. 44-48.
44. Rao, R. R., Gouthami, K. and Kumar , J.V. (2013) , “ Effects of Velocity-Slip and Viscosity Variation in Squeeze Film Lubrication of Two Circular Plates”, Tribology in Industry, Volume 35(1), pp. 51- 60.
45. Reynolds, O. (1886), “On the theory of lubrication and its application to Mr. Beauchamp tower’s experiments, including experimental determination of the viscosity of olive oil”, Philosophical Transaction of the Royal Society, Vol. 177, pp. 157-234.
46. Shekhar, K., Saxena, A. and Tyagi, A. P. (2016), “Mathematical Modeling of Diseased Human Knee Joint under Jumping Condition”, Proceeding of the 10th INDIACOM-2016, BVICAM (New Delhi), pp.5545-5549.
47. Shekhar, K., Tyagi, A. P., Saxena, A. and Kumar, D. (2014), “Analysis of an Exponential Slider Bearing Under Highly Loaded Conditions of Human Knee Joint: Pure Rolling Case”, International Journal of Applied Engineering Research, Vol. 9, No. 21, pp. 8543-8553.
48. Sinha , P. and Adamu, G. (2009), “THD analysis for slider bearing with roughness: special reference to load generation in parallel sliders”, Acta Mech. 207, pp. 11–27.

49. Shukla, J.B. (1978), “A new theory of lubrication for rough surfaces”, Wear, Vol. 49, pp. 33-42.
50. Spikes, H. A. (1994), “The behavior of lubrications in contacts: current understanding and future possibilities”, J. Porc. Instn. Mech. Engrs., Vol. 28, pp. 3-15.
51. Steindler, A. (1955), “Kinesiology of the human body”, Charles C. Thomas Publishers, springfield Illinois.
52. Tyagi, A. P., Saxena, A. and Shekhar, K. (2010), “Mathematical Modeling of Normal Human Knee Joint Function Under Highly Loaded Conditions: One Layer Model: Effects of HA Molecules under Squeezing (Symmetrical Case),” International Journal of Applied Mathematics and Applications, Vol. 2, No. 2, pp. 183-195.
53. Tyagi, A. P., Saxena, A. and Shekhar, K. (2010), “Mathematical Modeling of Normal Human Knee Joint Function Under Highly Loaded Conditions: One Layer Model: Effects of HA Molecules under Squeezing (Non- Symmetrical Case),” International Journal of Mathematics and Applied Statistics, Vol. 1, No. 2, pp. 127-138.
54. Tortora G. J., and Grabowski S. R., (2004), “Introduction to the human body: The essentials of anatomy and physiology”, John Wiley & Sons, Sixth Ed., USA, pp. 619.
55. Wang, M. L. and Peng, Z. X. (2015), “Wear in human knees”, Biosurface and Biotribology I, pp. 98–112.
56. Wright, V. and Dowson, D. (1976), “Lubrication and cartilage”, J. Anat., Vol. 121(1), pp. 107-18.
57. Yang, T. (2003), “A three-dimensional biphasic finite element formulation for hydrated soft tissue”, PhD Thesis, Mechanical, Aeronautical and Nuclear Engineering, Rensselaer Polytechnic Institute. pp. 164.